THE DEVELOPMENT OF A PERSONALIZED SYSTEM OF INSTRUCTION (PSI) FOR PRE-CALCULUS MATHEMATICS (MA 1021)

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Monterey, California

THESIS

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by

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Submitted in partial fulfillment of the
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MASTER OF SCIENCE IN MANAGEMENT

ABSTRACT

There is a vast opportunity to apply the Personalized System of Instruction technique within the military atmosphere, and especially at the Naval Postgraduate School where the emphasis is on graduate education. The subject of this thesis is to explore the Personalized System of Instruction method and to develop a pre-calculus course using this method for use at the Naval Postgraduate School.

TABLE OF CONTENTS

I. INTRODUCTION

One of the major emphases permeating the history of mathematics education, and indeed education in general, is the emphasis that has been placed on developing the capabilities of the individual student and the concomitant tendency to place upon him considerable responsibility for his own intellectual development. Although all students have different intellectual abilities, and although aptitude is a factor, success in mathematics and in other curricula is usually dependent upon the student's previous experiences, which are indeed varied. Due to the group-oriented method of instruction that has been traditional in the American system of education, many students have gaps in their educational backgrounds. These gaps, or areas of weakness, prevent the student from fully reaching the level of educational maturity and competence which is commensurate with his ability. An individualized or personalized program of instruction is designed to diagnose and eliminate these apparent gaps, and to limit the creation of future gaps in the student's knowledge.

A. BACKGROUND

If the proposition that learners are unique as individuals is accepted, then such areas as instructional techniques, text development and selection, grading, and other educational considerations should be predicated upon it. Some experienced educators suggest that tailoring education to the unique qualities of the individual is desirable but, as yet, idealistic. Others would argue that traditional instruction should

be preserved, or that costs have always made individualization impractical, neither assumption being proven. The real world of education is one of compromise, and the availability of an individualized program for students has only been realized in recent years.

Bloom [1] and Tyler [2] have shown that the environment has its greatest influence on a child's development in the first years of his life, and that thereafter, the influence of the environment stabilizes and becomes predictable. Long before children enter school they have learned a variety of motor skills; discovered verbal and non-verbal communication techniques; and been made aware of attitudes and values. It is noteworthy that they learn these things individually rather than in a group, and without the discipline and regimen of the formal school setting.

As a consequence of this preschool learning process children come to school needing instruction suited to their individual abilities. The teacher of these children can utilize the group-oriented process as the model of instruction, or the teacher can build upon the student's enthusiasm and interest and encourage their individual initiative, and provide a variety of experiences that challenges each child at his own level of ability.

B. GROUP-ORIENTED INSTRUCTION

Group-oriented instruction is the traditional method of instruction used throughout the American educational system. It has not been particularly effective as evidenced by the large numbers of dropouts and non-readers who graduate from high school. Group-oriented instruction has also proven to be inefficient due to the amount of re-

learning that is necessary each year as students progress through the educational system and instructors at each level reteach portions of the subject matter covered at lower levels.

Recently, the traditional approach of teaching to group standards has been giving way to an individualized approach to education wherein mastery is measured by absolute standards set at attainable levels in the light of a student's present educational development [3]. That is, the question put to the student is "How soon can you perform this task at this specified level of proficiency?" rather than "Can you perform this task as well as, and as soon as, your classmates?" The first standard is set largely as a function of the mastery needed for further study, while the second standard is mostly a function of class composition, which can change independently of the individual's own efforts, accomplishments and abilities.

The traditional method of instruction has encouraged students to compete: "Pass if you can or fail if you must." When students do fail, then another problem arises, for passing to the next grade level without their having earned it becomes a social necessity and denies students a sense of accomplishment and encourages further mismatches between their own abilities and those of their classmates. Bloom [3] has shown that a teacher who begins a new term feeling that one-third of his students are going to fail, when coupled with supportive school policies and grading systems that enable him to transmit this feeling to the students, creates a self-fulfilling prophesy; the final sorting of students through the grading process becomes approximately equivalent to the original expectations.

C. INDIVIDUALIZED LEARNING

According to Gagne [4], modern learning theorists consider learning to be a change that takes place inside the learner. The change is not in the stimulus, or in the immediate effects, or in the response; but it is a change that consists of a rearrangement of internal neural processes.

One implication of this theory is that educational writers will have to become more precise in their target - the individual learner. A second implication of modern findings, and the learning model that describes them, is the unmasking of the delusion that learning is in some manner or other an event of social interaction. Gagne [4] states that there is little truth to this argument except in quite a secondary sense. Learning is an individual act, a set of events which takes place entirely within the learner. And, in fact, it is a process which depends very much on the nature of the learner, particularly his own past learning. The learner is, then, in a fundamental sense responsible for his own learning.

D. THE PERSONALIZED SYSTEM OF INSTRUCTION

Programmed instruction, with its emphasis on incremental learning and demonstrated step-by-step mastery toward clearly-defined goals, marked the introduction of the personalized system of instruction (hereafter referred to as PSI). Glaser [5] has defined this personalized approach as the adaptation of instructional procedures to the requirements of the individual learner. However, in the generic sense, the PSI method in education implies a tailoring of the educational process which takes into account the particular qualities and needs of each student.

Keller [6] summarized the features of PSI which distinguish it most clearly from the traditional group-oriented approach:

1. <u>Self-pacing</u>

   The go-at-your-own-pace feature allows a student to cover the course material and complete the course at a rate commensurate with the student's individual ability.

2. <u>Unit Mastery</u>

   The unit-perfection requirement permits the student to advance to new material only after he has demonstrated mastery of previous material.

3. <u>Motivational Lectures</u>

   Lectures and demonstrations serve as a means of motivation rather than as sources of critical information. Lectures can be used to present topics of special interest or to delve more deeply into some portion of the primary course material, and students should not be deprived of this experience. They are provided only for enrichment purposes, and only to students who have completed the work necessary to understand them.

4. <u>Essential Matter</u>

   The technique allows the teacher to present material to the student in writing, on tape, on film, by computer, or by any means accessible to the student when he is ready for it.

5. <u>Proctors</u>

   The use of proctors permits repeated testing, immediate scoring, available tutoring, and a marked enhancement of the personal-social aspect of the learning process.

Keller described the similarity between the PSI method and the programmed system of instruction by arguing that there is the same emphasis upon analysis of task, the same concern with terminal performance, and the same opportunity for individualized progression, and so on. But the principal steps in the process of advancement are different in the two systems. Under programmed system of instruction the student advances through completion of frames in a set, while under the PSI method the student progresses by completing units that are more like the conventional homework assignment or laboratory assignment. Keller [6] summarizes this difference by stating,

> "the 'response' is not simply the completion of a prepared statement through the insertion of word or phrase. Rather, it may be thought of as the resultant of many such responses, better described as the understanding principle, a formula, or a concept, or the ability to use an experimental technique. Advance within the program depends on something more than the appearance of a confirming word or the presentation of a new frame; it involves a personal interaction between a student and his peer, or his better, in what may be a lively verbal interchange, of interest and importance to each participant. The use of a programmed text, a teaching machine, or some sort of computer aid within such a course is entirely possible and may be quite desirable, but it is not to be equated with the course itself."

E. THE PSI METHOD FOR PRE-CALCULUS

There is a vast opportunity to apply PSI within the military atmosphere, and especially at the Naval Postgraduate School (NPS) where the emphasis is on graduate education. The use of PSI at NPS to refresh students who have been away from the educational environment, or even in graduate work itself, is appealing.

The design of a set of instructional materials for use in teaching a pre-calculus course under the PSI method of instruction is developed

in chapter two. The course, which is presented in chapter three, is designed to allow students to become more independent of the instructor, thereby becoming more self-reliant, responsible, and involved in the learning process. The course materials consist of a text and a study guide. The study guide is divided into three units and each unit contains a pretest for the student to self-administer prior to taking the mastery test for the unit. Each unit also contains a rationale and a set of objectives to be accomplished prior to taking the pretest. These materials furnish a customized educational sequence for the individual, allowing him selection from a variety of learning opportunities. Chapter four deals with methods for testing the effectiveness of PSI.

II. DEVELOPMENT OF A PSI STUDY GUIDE

If the objective of the educational system is to institute a continuous learning program in which the learning process is student centered, highly individualized, and increases the opportunities for students to accept responsibilities for and maintain an active role in their learning, then the PSI method is ideal.

PSI is not a passive approach, not a correspondence course, not merely a substitute for lectures. Rather, PSI is a method by which decisions arrived at through concerted planning are executed. It permits the instructor to plan in terms of expected behavioral changes. It allows the student to become much more active in his learning process. It allows both the instructor and the student the flexibility necessary in a well-developed education environment. PSI is a method conceived and implemented through a mutual desire to improve the educational product. As a group, instructors that are attempting to personalize their instruction accept the principle that all learners can be taught better if they are not considered "all" learners, but rather as "individual" learners. In this chapter the strengths and weaknesses of PSI are discussed, and subsequently, the design for a PSI study guide and the military interest in PSI are explicated.

A. PSI STRENGTHS

PSI is not a panacea for the educational system, but is perhaps the most widely and intensively investigated method of all recent attempts to improve teaching (7). PSI is rooted in learning reinforcement theory and, therefore, one of PSI's most salient advantages is that it makes

failure visible at a time and under conditions when something can be done to correct it. The basic law of reinforcement theory is Thorndike's Law of Effect which states, "a response followed by reinforcement increases its frequency." As Gallup (8) points out, the reinforcement contingencies can be experimented with. On a large scale this is not true for conventional instructional techniques.

Under the PSI technique the student plays a much more active role as opposed to the passive role established in group-oriented instruction. The use of challenging problems that require students to make decisions, followed by an indication to them of how often they are correct, can make each step in the learning process an intrinsically rewarding one.

The instructor is not eliminated in the PSI procedure nor are the students abandoned to teach themselves as in a correspondence course; PSI is not a correspondence course. In fact, the instructor is much more involved than in a conventional course, and spends a considerable amount of his time in the learning environment under the PSI method. The instructor becomes a "contingency manager" and writes a program for learning. Additionally, and this is an indispensable part of the PSI technique, he is available not only to oversee the learning environment but to answer questions, to treat the unique, and to add the artistry that cannot be programmed. The PSI system unshackles the instructor from addressing the usual, and permits him the time to deal with the creative.

Proctors are available to evaluate each student's work immediately, face-to-face, and to provide personal tutoring, counseling, and encouragement. Proctors provide a type of reinforcement, however subtle

and ill-defined, which cannot be built into a programmed text.

PSI is a plan, a suggested procedure for the implementation of a type of learning that hopefully will achieve the objectives of the educational system.

B. PSI WEAKNESSES

The tenor presented thus far for PSI borders on unrestrained abandon. However, PSI is probably not appropriate, possible, or even desirable for a certain portion of the academic curricula. Keller (9) and Green (10) state that the PSI method should not be used in any course where subject matter mastery is less important than regular exposure of the student to some charismatic figure by means of lectures or seminars.

Second, the method will not serve the instructor who believes the student should control the content of the course and the degree to which it is understood.

Third, if the subject matter changes rapidly or there is not an adequate text available for the course then the PSI method should be avoided. If the course cannot be predicted in advance and text materials are not available, then it is impossible to plan a coherent course and study guide for the course.

Fourth, PSI is not suggested as a solution to the problem of excessive numbers of students. The problem here is one of proctors and educational control. To how many proctors can the instructor give guidance and the attention necessary to assure their optimal performance day by day? In other words, does the scheduled proctors-instructors meeting become unwieldy leaving questions to be unanswered, mistakes to go by undetected, reports on students to be unheard, and the general

exchange of experience and information to be lost? Presently there is no clear answer to such questions, however, it appears that more than ten or twelve proctors are difficult to manage efficiently (9) which would limit the class size to 100-200 students.

Finally, PSI may not be for the instructor who feels he is currently successful, and that the traditional practices are basically sound. Green (10) feels this is the best reason not to try the PSI method since without an enthusiastic desire to make it work, it probably will not.

C. THE PSI STUDY GUIDE

The PSI Study Guide is designed to allow students to become more independent of the instructor, more self-reliant and more involved in the learning process. It furnishes a customized educational sequence for the individual learner and allows him to select from a variety of learning opportunities via media, resources, and activities. Since a diverse range of student abilities are dealt with, numerous paths should be provided for the students to exhibit these abilities. In PSI there are basic ingredients common to every study guide prepared for a course.

1. <u>Introduction or Rationale</u>

The introduction or rationale portion explains why the students are studying the concept and also provides motivation to study the concept. It attempts to relate those experiences which the students have encountered previously to those which they will presently be encountering, and also to relate the present experiences to ones they will encounter in the future. The rationale should be aimed at the student in his language and should be more than just a collection of

educational morsels to impress the intellectual. The rationale is an effective selling job answering the poignant query, "What's in it for me?"

2. Objectives

The purpose of instructional objectives is to clarify to the students what is expected of them. More simply, the objectives should fulfill the essential function of identifying for the student precisely what he or she is to learn and the kind of performance necessary to show that he has learned the material.

Stating the objectives of a course allows the instructor to plan instruction in terms of expected behavioral changes. The behavioral or performance goals then become the focal point of the educational process. By necessity, the planning of a PSI course becomes student centered and more specific. Activities are organized so that students are presented with problems whose solutions require an understanding of the basic concepts under investigation.

The ability to evaluate the success of teaching is one of the most valuable aspects of designing a course around objectives. It permits modification of the program and improvement of instruction.

A course written in terms of objectives lends itself to maximum motivation, which is certainly desirable. The students are aware of just what is expected, and therefore, use their time in a more efficient manner (since it is easier to work toward a goal that is explicitly specified).

Green (11) gives three benefits for stating explicitly to the student and objectives of the course:

a. The student may already know the material and can save

time once he knows what you are trying to accomplish.

b. The student may have a better idea of how to accomplish the objective.

c. When the student recognizes that he has reached the goal he receives a personal satisfaction of having accomplished something; this is very motivational.

3. Procedure

The purpose of the procedure section is to tell the student how to learn the material and other matter referred to in the section. According to Green (11), this section is the most difficult to write because you are not aware in advance of how much difficulty the students will encounter trying to master the material contained in the unit.

4. Self-Test

The self-test is used to indicate to the student whether he has learned the material. It is self-administered and the correct solutions are available in the study guide for the student to check his answers. This permits him to see if the objectives of the unit have been met or if specific skills are lacking and require more work and review. The self-test establishes for the student the decision point where he decides that he has or has not mastered all the learning objectives. If the student feels he has mastered the objectives of the unit, then he proceeds to take the mastery test. If it is successfully completed, then the student can proceed to the next unit and begin the process again.

In summary, the PSI study guide outlines the PSI course as a highly structured program of learning material with three essential and unique aspects.

a. The instructor is built into the course as an integral and indispensible part. The instructor now fills the roles of guidance person, human relations person, resource person and, to a lesser degree, dispenser of information and evaluation.

b. Opportunities for the student to make choices are built into the course. The student may choose to take the self-test if he thinks he already has mastered the objectives, or he may choose which assignments to do for a specific objective within the general framework of those offered; or he may choose to work to a greater depth on the course if so desired.

c. The components of a PSI course are highly structured and, at first thought, a person would say that highly structured means little or no room for the individual and his special needs. But the reverse is true. The structure of the course depends on clearly-stated objectives or goals, which the student understands he must accomplish in order to progress.

D. MILITARY APPLICATION OF PSI

The opportunity to apply PSI within the military environment is as great as in civilian institutions of education. Specifically, at the Naval Postgraduate School (NPS), where the emphasis is on graduate education, the use of PSI to refresh personnel who have been away from the academic environment for a number of years, or provide the basics for those who are transitioning into graduate work, or indeed to graduate work itself, is particularly appealing. If one of the goals of NPS is to improve the quality of education it offers its students at a reasonable cost, then the PSI system could be utilized to accomplish

this objective by permitting more able or better qualified students to progress more rapidly through courses while still providing the same education to those for whom the material is new or for whom a slower pace is more comfortable. Students come to NPS possessing a wide range of backgrounds and abilities, and for this reason it is important that introductory courses be sufficiently flexible to meet the varied needs of these students. Since a wide range of student abilities are being dealt with at NPS, then many different pathways must be provided for all students.

PSI courses can be utilized to assist personnel prior to their arrival at NPS. Prospective students might partially complete courses before their matriculation at NPS, thereby expediting the learning process, reducing the time on board and consequently, reduce the cost involved for their education.

The study guide presented in Chapter three is intended for use in the pre-calculus mathematics course at NPS. It incorporates the basic ingredients of a PSI study guide described above in Section C. It is appropriate for use either on-campus or off-campus.

III. THE PSI STUDY GUIDE FOR PRE-CALCULUS MATHEMATICS

The study guide presented in this chapter is keyed to the text, *Fundamentals of College Algebra and Elementary Functions*, by Johnson, Lendsey, Slesnick and Bates, and has been prepared to facilitate the teaching of pre-calculus mathematics. The study guide divides the course into four units of approximately equal length. Each unit contains its own introduction (rationale), objectives, procedure, summary and self-test.

TEXT

FUNDAMENTALS OF COLLEGE ALGEBRA AND ELEMENTARY FUNCTIONS

by

Johnson, Lendsey, Slesnick and Bates

PREFACE

This study guide has been prepared to facilitate the teaching of pre-calculus by the PSI or Keller method of instruction. This method differs from the traditional lecture-recitation method in that it permits the student to progress through the material at his own pace and to check his mastery of the subject at frequent intervals. The PSI method of instruction and this study guide have been especially designed for you, the student.

The course is divided into four units. Each unit begins with an introductory or rationale paragraph which briefly describes the subject matter contained in that unit. Next, there is a list of learning objectives which clearly define your responsibilities. You should read the objectives carefully because each unit test is designed to measure how well you have mastered the skills specified by the learning objectives. After the objectives comes the Procedure section which outlines the reading assignments and assigns exercises in the text to be completed. The Procedure section of the study guide describes the specific procedures you must follow in order to master the material contained in that unit and meet the learning objectives. Following the Procedure section is a summary of the important mathematical concepts and skills covered by that unit. This section reiterates important

points and assigns review problems that will enable you to check your understanding of the material prior to taking the self-test and the unit mastery test. The self-test is a short test which serves as a review for your mastery of the material immediately preceding the unit mastery test.

You may work on these units whenever and wherever you like. You should work through this material at your own pace, proceeding quickly through those units that contain subject matter that is familiar to you or that you find easy to master, and working more slowly through those units that present material that you find more difficult.

MA 1021 PRE-CALCULUS MATHEMATICS

PSI STUDY GUIDE

TABLE OF CONTENTS

Unit 1

Fundamentals of Algebra and the Number System

Introduction

Have you ever tried to define the word "number"? If you have, you probably said it had something to do with the idea of "how many." However, the idea of "how many" is only partly correct because many kinds of numbers are assigned different meanings. Numbers are abstractions and we give them meanings that we want them to have.

In this unit we shall study the number system used in mathematics and the development of the algebraic structure of the real number system. Since much of mathematical development has its roots in practical requirements and applications, we shall use these concepts of algebra to evaluate and understand formulas, analyze relationships and solve verbal problems. When the facts of a problem are translated into the language of algebra, we can focus our attention on the relevant details and reduce the problem to that of routine operations.

Objectives

When you have mastered this unit you should be able to:

1. Define and give examples of the following terms:
    - a. real number
    - b. integer
    - c. rational number
    - d. irrational number
    - e. algebraic expression (mononomial, binomial, trinomial, & polynomial)
    - f. variable

g. absolute value

h. coordinate

i. imaginary number

j. complex number

k. discriminant

l. conjugate of a complex number.

2. Describe the interrelationships of the various number systems and the concept of closure within the system. (See chart in Summary section of this unit).

3. Combine and simplify fractional and non-fractional algebraic expressions using addition, subtraction, multiplication, division, and factoring.

4. Multiply and divide algebraic expressions utilizing the four "Laws of Exponents."

5. Define a^n whenever n is a positive integer or 0.

6. Define a^n whenever n is a negative integer.

7. Define a^n whenever n is a rational number.

8. Solve problems involving radical expressions by utilizing the "Laws of Radicals" and by "rationalizing the denominator" such as those given in the exercise section.

9. Describe what is meant by the real number line and its connection with the real number system.

10. Define the concept and be able to solve problems involving absolute value such as those given in the exercise section.

11. Define the concept of imaginary numbers and solve problems involving complex numbers and their conjugates such as those given in the exercise section.

12. Solve any real quadratic equation using the quadratic formula; identify the discriminant of a quadratic solution; and state three cases that determine the condition of the roots by examining the discriminant.

13. Graph quadratic equations on the Cartesian Coordinate system.

Procedure

1. Read sections 1-1 through 1-3 in the text. Read this material twice through - once quickly for an overview of the ideas and then for a second time carefully, making sure you can define the terms "integer," "rational number," "irrational number," "simplest form of a rational number a/b" and "closed system."

Then solve problems:
a) 5, 7, 9, 13, 15, 19, 21, 25, and 29 of exercises 1-2.
b) 1, 11, 13, and 15 of exercises 1-3.

2. Read sections 1-4 through 1-8 in the text. Read it twice through - the first time quickly for an overview and then again carefully, paying particular attention to the "Four Laws of Exponents." Work through the sample problems that illustrate examples of these four laws. Be able to give definitions for "variable" and for "algebraic expression" in section 1-4, and for the definition of "x^0" in section 1-8.

Solve problems:
a) 3, 9, and 15 of exercises 1-4.
b) 1, 3, 5, 9, 17, 19, 21, 23, 25, 31 and 37 of exercises 1-5.
c) 1 - 15 (odd) of exercises 1-6.
d) 5, 7, 9, 11 of exercises 1-7.
e) 7, 9, 11, 13, 15, 23 and 25 of exercises 1-8.

3. Read sections 1-9 through 1-12. Read it twice through - the first time quickly to get an overall idea of the concepts to be covered in these four sections, then read it a second time in more detail.

In section 1-9 be able to define the "nth root of a number," "index," and be able to use the "Laws of Radicals" to simplify or combine algebraic expressions.

In section 1-11 read through the method used to eliminate radicals in the denominator called "rationalizing the denominator" and in section 1-12. Be able to give the definition for "rational exponents."

In all four sections follow through the sample problems with the author and be able to explain why each step has occurred. These sample problems are applications of the material being covered in the section.

Solve problems:
a) 1, 3, 5, 7, 11, 13, 17 and 23 of exercises 1-9.
b) 1, 3, 5, and 7 of exercises 1-10.
c) 1, 3, 7, 9, 11, 13, 17 and 18 of exercises 1-11.
d) 1, 3, 7, 9, 11, 13, 15, 17, 21, 29 and 31 of exercises 1-12.

4. Read section 1-13 twice, the first time for an overview and then again more carefully. Be able to expound on the components that make up the number line and how the number line is used in connection with the real number system. Be able to define such terms as "coordinate," "origin," and "directed distance."

Memorize the definition of "absolute value" and be able to state in writing, the two properties of the concept of "absolute value"

keeping in mind that the absolute value of every nonzero number is a positive number (refer to pp. 36 in text).

Solve problems: 1, 3, 5, 7, 9, 11, 13, 15, 19 and 23 of exercises 1-13.

5. Read sections 9-1 and 9-2 twice through - once for an overview and again in detail. Be able to define the concept of imaginary and complex numbers. Know the definition for a complex number and an imaginary number in section 9-1.

In section 9-2 be able to eliminate "i" from the denominator of a given quotient by using the "conjugate" of the denominator. Be able to find the conjugate of a denominator.

Follow through the sample problems with the author in section 9-1 and section 9-2 to get a better understanding of complex numbers and their application in specific problems.

Solve problems: a) 1, 5, 9 and 15 of exercises 9-1.
b) 1, 3, 5, 15, 17 and 19 of exercises 9-2.

6. Read sections 9-4 and 9-5 twice through - once for the overview of the quadratic formula and how to graph a quadratic equation, and then a second time for more detail and insight into these concepts.

Know the meaning of the term "discriminant." Be able to determine the character of the roots of a quadratic equation from the discriminant. Be able to solve any real quadratic equation in the form $ax^2 + bx + c = 0$. Follow through in step-by-step fashion the author's sample problems on quadratics.

Be able to graph a real quadratic function. Follow through the example problems and see how the three cases of the discriminant determine the outcome of the graph.

Solve problems: a) 1, 3, 5, 11, 15, 19 and 21 of exercises 9-4.

b) 1, 3, 5, 7, 9 and 11 of exercises 9-5.

7. Read the Summary paragraph, then take the self-test to see if you have mastered the objectives of this unit. If you have any questions or uncertainties, check with your proctor. When you have completed the self-test ask your proctor to administer the Unit 1 Mastery Test.

Summary

In this unit you have studied various number systems, how they are structured and how it is integrated with the number line. You have learned the definitions of many terms in the algebraic system and through problem-solving, you should have come to a clearer understanding of these terms. Your work in this unit will make it easier for you to move ahead into more difficult areas knowing that you have a good foundation on which to build.

Now that you are aware of the importance of the number systems to algebra, the following diagram should summarize the interrelationships of these number systems.

Unit 1 Self-Test

1. Find the sum of each of the following pairs of numbers. Leave answers in radical form.

a. $3\sqrt{7} + 2, \quad 2\sqrt{7} - 6$ b. $14 + \sqrt{33}, \quad 2 - 3\sqrt{33}$

c. $2\sqrt{5} + 3\sqrt{8}, \quad \sqrt{5} - 2\sqrt{8}$ d. $6\sqrt{3} + 4, \quad 2 + 3\sqrt{6}$

2. Peform the indicated operations.

a. $(3 - 2\sqrt{3})\cdot(3 + 2\sqrt{3})$ b. $(11 + \sqrt{2})\cdot(1 + \sqrt{2})$

c. $(4x + 2y - 1)+(-3x - 4y + 2)$

d. $(4x - 17y + 12)-(7x + 14y - 18)$

3. Find the following products and quotients.

a. $(3x\,y^2\,z^3)(2x^3\,y^4\,z)$ b. $(17x^3\,z^2)(-2x^4\,y^5)$

c. $5a(2a + 3b)$ d. $(8x - 3y)(x - y)$

e. $(18x^5\,y^3) \div (6x^3\,y^2)$ f. $(14x^4\,y^2\,z^3) \div (7x^3\,yz^2)$

g. $(3x^4\,b^6 + 2x^3\,b^5) \div x^2\,b^3$

4. Find the following sums and differences.

a. $\frac{3}{13} - \frac{1}{2}$ b. $\frac{x}{y} + \frac{3}{y^2}$

c. $-3 + \frac{6}{x} - \frac{2}{x^2}$

5. Simplify the following:

a. $4^2 \cdot 4^0 \cdot 4^1$ b. $3^6 \cdot 3^{-3} \cdot 3^0$

c. $(3x^{-2}\,y^{-2})(2x^5\,y^4)$

d. $\frac{2x^3y}{x^5y^3}^{\,3}$ e. $\sqrt{48}$ f. $18x^5y^3$

g. $\frac{16x4}{y^6}$ h. $\frac{x^4}{12y^2}$ i. $4xy^2 + y^3$

j. $2x^3y^4 - 12x^2$ k. $\sqrt{48} \cdot \sqrt{96}$

l. $3x \cdot 2x^3$ m. $2^{2/3} \cdot 4^{2/3}$

n. $(x^{1/3} y^{2/3})^4$ o. $\frac{a^4y^2}{16a^2y^2}$

6. Rationalize the denominator and simplify.

a. $\frac{2\sqrt{6} + 4}{2\sqrt{6} - 3}$ b. $\frac{\sqrt{13} - 2\sqrt{3}}{3\sqrt{13} + 6\sqrt{3}}$

7. Perform the indicated operations.

a. $|15 - 17| - |7 - 5|$ b. $|\frac{2}{11} - \frac{4}{7}|$

c. $|10 - 4| + |3 - 5|$ d. $|\frac{1}{5} - \frac{8}{25}|$

8. Find all integral values for x for which each expression below is true.

a. $|x + 7| = 19$ b. $|x - 2| < 18$ c. $|x - 3| \leq 9$

9. Perform the indicated operations.

a. $(i\sqrt{3})^3$ b. $(-17 + i\sqrt{3}) - (4 - 3i\sqrt{3})$

c. $\frac{12+i}{2-i}$ d. $\frac{2-i\sqrt{3}}{2-3i}$ e. $(3+2i)(1-5i)$

10. Solve the following quadratic equations

a. $2x^2 + 8x + 14 = 0$ b. $x^2 - 4x - 5 = 0$

c. $x^2 - 4x + 4 = 0$

11. Graph the quadratic equations.

a. $f(x) = x^2 - 4x - 5$ b. $f(x) = x^2 - 4x + 4$

12. Write a quadratic equation for each of the following solutions sets.

a. $\{3i, -3i\}$ b. $\{2-2i, 2+2i\}$ c. $\{3, -5\}$

Answers to Unit 1 Self-Test

1. a. $5\sqrt{7} - 4$ b. $16 - 2\sqrt{33}$ c. $3\sqrt{5} + \sqrt{8}$ d. $6 + 6\sqrt{3} + 3\sqrt{6}$

2. a. -3 b. $13 + 12\sqrt{2}$ c. $x - 2y + 1$

d. $-3x - 31y + 30$

3. a. $6x^4y^6z^4$ b. $-34x^7y^5z^2$ c. $10a^2 + 15ab$

d. $8x^2 - 11xt + 3y^2$ e. $3x^2y$ f. $2x^2y\ z$

g. $3x^2b^3 + 2xb^2$

4. a. $-2/13$ b. $\frac{xy + 3}{y^2}$ c. $-3x^2 + 6x - 2$

5. a. $6x$ b. 27 c. $6x^3y^2$ d. $\frac{8}{x^6y^6}$

e. $4\sqrt{3}$ f. $3x^2y\sqrt{2xy}$ g. $\frac{4x^2}{y^3}$ h. $\frac{x^2}{2y\sqrt{3}}$

i. $2y\sqrt{x} + y\sqrt{y}$ j. $xy^2\sqrt{2x} - 2x\sqrt{3}$ k. $48\sqrt{2}$

l. $x^2\sqrt{6}$ m. 4 n. $x^{4/3}y^{8/3}$ o. $\frac{a}{4}$

6. a. $\frac{36+14\sqrt{6}}{15}$ b. $\frac{1-4\sqrt{39}}{7}$

7. a. 0 b. $\frac{30}{77}$ c. 8 d. $\frac{3}{25}$

8. a. $\{-26, 12\}$ b. $\{-16, 20\}$ c. $\{-6, 12\}$

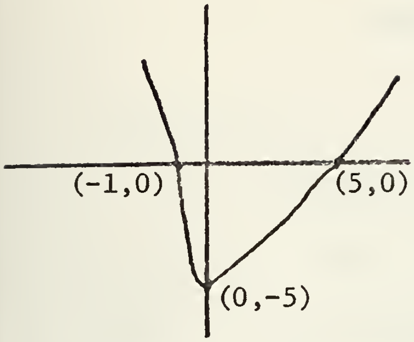
9. a. $-3i\sqrt{3}$ b. $-21 + 4i\sqrt{3}$ c. $\frac{23 + 14i}{5}$

d. $\frac{4+6i - 2i\sqrt{3} + 3\sqrt{3}}{13}$ e. $13 - 13i$

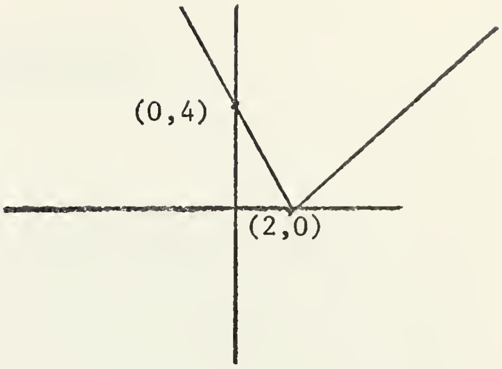
10. a. $x = \{-2 \pm i\sqrt{3}\}$ b. $x = \{5, -1\}$

c. $x = \{2\}$

11. a.



b.



12. a. $x^2 + 9 = 0$ b. $x^2 - 4x + 8 = 0$ c. $x^2 + 2x - 15 = 0$

Unit 2

Systems of Equations and Inequalities

Introduction

The principal topic of algebra over the years has been solving equations. Today, inequalities have also been studied because of their importance and application to other sciences. Our main purpose in this unit is to provide a logical basis for solving equations and inequalities. We shall assume that we are seeking solutions for the set of real number, and therefore we will be able to use the properties of the real number system. Another purpose of this unit is to illustrate how the properties of the real number system are involved in the process of solution, and to present practical applications for the solutions.

Objectives

When you have mastered this unit you should be able to:

1. Define and give examples of the following:
    a. first-degree equation (linear equation)
    b. solution set of an equation
    c. equivalent equations
    d. first-degree inequality (linear inequality)
    e. solution set of an inequality
    f. equivalent inequalities
    g. intersection, union and empty set
    h. equation in two variables
    i. the graph of the set of ordered pairs
    j. slope of y-intercept of a line
    k. inequalities in two variables

    l. graphs of inequalities using related equations
    m. a system of equations in two variables
    n. inconsistent equations; dependent but consistent equations; and independent and consistent equations.

2. Solve equations by using equivalent equations in the solution process.

3. Solve inequalities by using related equations.

4. Apply linear equations in one variable to solve word problems.

5. Graph an equation and inequality, and conversely, be able to write the equation or inequality if given the graph.

6. Graph two equations and describe their intersection.

7. Graph two inequalities and describe their intersection and union.

8. Find the slope of a line.

9. Given two points, find the equation of the line passing through these points.

10. Solve equalities and inequalities in two variables.

11. Be able to solve systems of equations in two variables using the "substitution" and the "addition" methods.

12. Determine whether a system of equations is independent, dependent, or inconsistent.

13. Apply your knowledge to solve systems of equations and inequalities using the method of related equations and the method of equivalent equations.

<u>Procedure</u>

1. Read sections 2-1 through 2-3 in the text. Read this material twice through - once quickly for an overview and then a second time

carefully making sure you can define the terms "first-degree equation," "solution set," "equivalent equations," "first-degree inequality," "equivalent inequalities," "intersection," "union," and "empty set." Know the meaning of the relation signs in section 2-3 and be able to use them in problem-solving in the assigned exercises.

Follow through the sample problems in each section in a step-by-step fashion to gain a clearer understanding of the material presented in the particular section and to see practical application for the material.

Solve problems: a) 1, 3, 7, 9, 13, 15 and 21 of exercises 2-1

b) 1, 3, 5, 9, 11, 13, 17 and 21 of exercises 2-2

c) 1, 3, 5, 7, 11, 13, 15, 17, 19 and 21 of exercises 2-3.

2. Read sections 2-4 through 2-6 in the text. Read the material once quickly for an overall view of the subject matter presented in these three sections, then read the material a second time in more detail seeing how this material has been extended from what you learned in the previous sections. Be able to use the Cartesian Coordinate system described in section 2-5, and the "graph of the set of ordered pairs" that is developed. Also be able to explain the concept of "slope" and how it is determined in the coordinate system. Know the definition of "y-intercept" and how it is used.

In section 2-6 the determination of the slope of a line using the "slope-intercept" form of the equation is explored. Follow through the sample problems in sections 2-5 and 2-6 in a step-by-step fashion to

see just how the equation of a line using a combination of the slope, y-intercept, and two points is determined. The sample problems should give a clearer insight into this concept in order that you might then use this concept to solve problems involving lines in the assigned exercises.

Solve problems: a) 3, 5, 7, 9 and 11 of exercises 2-4
b) 1, 3, 5, 7 and 11 of exercises 2-5
c) 1, 3, 5, 7, 9, 11 and 13 of exercises 2-6.

3. Read section 2-7 on inequalities in two variables for an overview of the concepts. Then read it a second time in more detail and depth. Be able to define "first-degree inequality in x and y," and how the concept of the "related equation" is used to graph inequalities. Be able to use equivalent inequalities with an obvious solution set to solve linear inequalities with no obvious solution set. The three sample problems are an integral part of this section and should be carefully followed in a step-by-step fashion and understood.

Solve problems: 1, 3, 5, 7, 9, 11 and 13 of exercises 2-7.

4. Read sections 3-1 through 3-3 twice through - once for an overview and then again in more detail. Note in section 3-1 how a system of equations in two variables is solved using the "substitution" method and in section 3-2 a system is solved using the "addition" method. Be able to solve systems using these two methods. The text's sample problems will give you a clearer insight into this - follow them through, step by step.

Also be able to give definitions for "inconsistent equations," "independent and consistent equations," and "dependent but consistent equations" in section 3-1. Be able to discuss these three types of equations.

In section 3-3 the solving of word problems by applying the properties of systems of linear equations is explored. Follow through the sample problems in a step-by-step fashion just how word problems are developed. Be able to solve such problems as those given in the exercises.

Solve problems: a) 1, 3, 5, 7, 9, 11, 17 and 19 of exercises 3-1

b) 1, 5, 7, 9 and 11 of exercises 3-2

c) 1, 3, 5 and 7 of exercises 3-3.

5. Read section 3-4 on systems of inequalities in two variables for an overview of the concepts. Then read it a second time more carefully. The sample problems are an integral part of this section and should be followed through in detail to give a clear understanding of the graphing of inequalities in two variables. Be able to solve problems for systems of inequalities in two variables similar to those given in the exercises.

Solve problems: 1, 3, 9 and 13 of exercises 3-4.

6. Read the Summary paragraph, then take the self-test to see if you have mastered the objectives of this unit. If you have any questions or uncertainies, check with your proctor. When you have completed the self-test ask your proctor to administer the Unit 2 Mastery Test.

Summary

In this unit you have studied first-degree equations and inequalities and how they are integrated with the Cartesian Coordinate system. You have learned the definitions of many terms and, through problem-solving, you have come to a clearer understanding of these terms and of

the concepts involved. Your work in this area will make it easier for you to move on to other concepts in algebra.

Unit 2 Self-Test

1. Solve the following equations and inequalities by using equivalent equations and inequalities.

a. $3x + 4 < 2x + 6$ b. $4(3x+5) + 1 = 8x + 1$ c. $\frac{8y+12}{80} > \frac{3}{4}$

2. Write an equation for the following problems and then solve the problem.

a. Two boxes weigh 84 pounds, and one of the boxes weighs 20 pounds more than the other. How much does each weigh?

b. The sum of two consecutive even integers is 50. Find the integers.

c. The sum of three consecutive even integers equals four times the smallest integer. What are the three integers?

3. Graph the following.

$$\{x \mid x + 4 > 2\} \cap \{x \mid x - 3 < 0\}$$

4. Graph each of the following linear equations and tell what its slope and y-intercept are.

a. $3x + 2y = 1$ b. $2x - y = 4$ c. $x + 3 = 0$

5. Find the equation of the line given the following.

a. slope = $1/2$ y-intercept = -3

b. the points $(1,1)$ and $(3,2)$

6. Graph the following inequalities:

a. $x \geq 3$ b. $x + 2y < 3$ d. $y \leq 2x + 1$

7. Solve the following systems of linear equations. Determine if they are inconsistent, independent and consistent, or dependent but consistent.

a. $\begin{cases} x = y + 3 \\ 2y = 2x - 1 \end{cases}$ b. $\begin{cases} 3x + y = 18 \\ x = 6 - x_3 y \end{cases}$ c. $\begin{cases} x + 3 = y \\ 2y + 3x = 1 \end{cases}$

8. Determine the solution set of the following systems of inequalities and draw the graph.

a. $\begin{cases} y \geq 0 \\ y \leq 3 \\ 2y \leq x \end{cases}$ b. $\begin{cases} 2x - y \leq 6 \\ y \leq 4 - 2x \\ x \geq 0 \end{cases}$

9. For each of the following problems write an equation and then use it to solve the problem.

a. The sum of two numbers is -40, and their difference is -8. What are the numbers?

b. One number is eight more than four times the other and their sum is -2. What are the numbers?

c. A 24-foot board is cut into two parts so that one part is 4 feet longer than the other, how long is each part?

Answers to Unit 2 Self-Test

1. a. $x < 2$ b. $x = 5$ c. $y > 6$

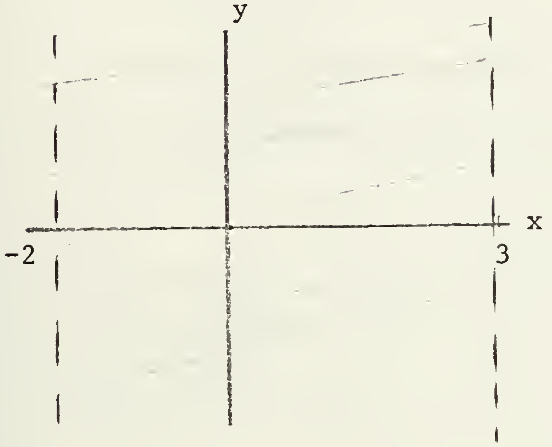
2. a. x = 1st box which weighs 30 lbs.

   2nd box weighs 30 lbs. and 20 lbs. more = 50 lbs.

   b. $x = \{24, 26\}$

   c. $x = \{6, 8, 10\}$

3.

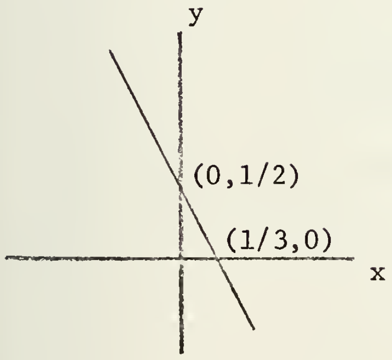


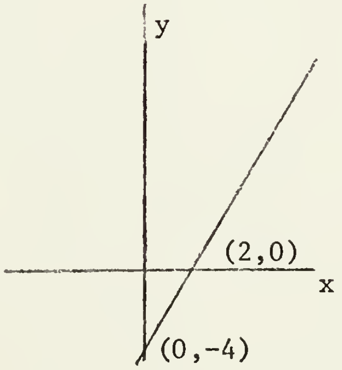
$x = \{x \mid x$ is the area between $x < 3$ and $x > -2\}$

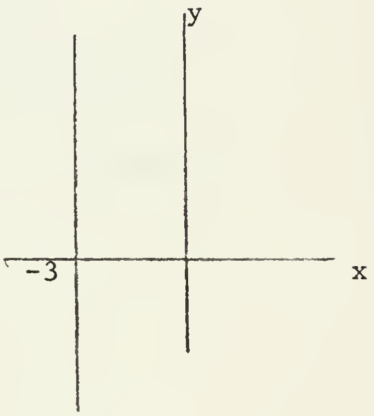
or

$x = \{x \mid -2 < x < 3\}$.

4. a. b. c.

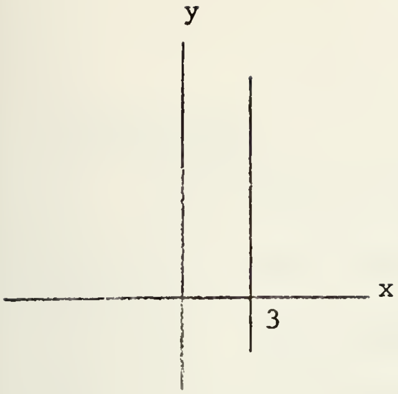






5. a. $2y = x - 6$ b. $2y = x + 1$

6. a.



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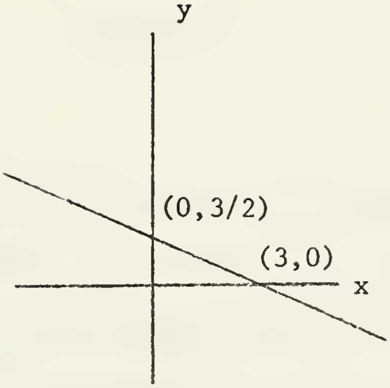

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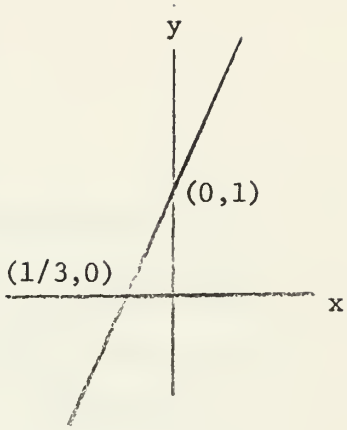
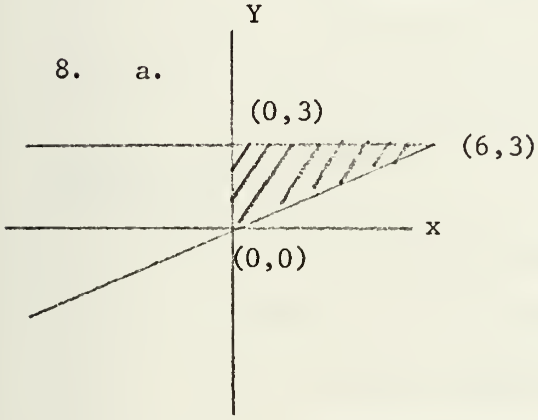


7. a. $\emptyset$; inconsistent

b. coincide; therefore dependent but consistent

c. $(-1,2)$; independent and consistent

8. a.



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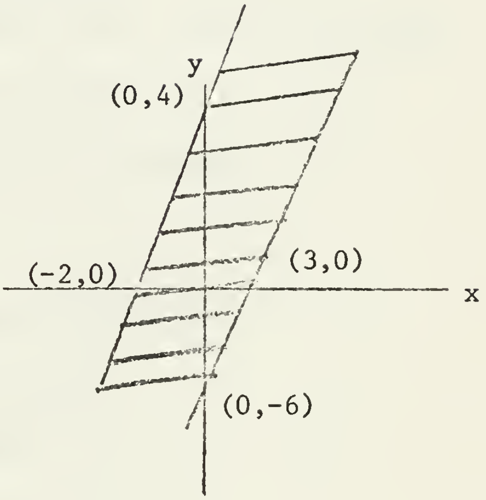
b.



PARALLEL STRIP

9. a. $x = \{-16,-24\}$ b. $x = \{0,-2\}$ c. $x = \{10,14\}$

Unit 3

Functions

Introduction

Surrounding us are numerous examples of changing quantities and variables, and in many cases there exists a relationship between these variables. The consideration of such relationships leads to one of the most important concepts of mathematics - the concept of "function." Functional relationships are important in many areas and are especially useful in the biological, physical, and social sciences. However, application of functional relationships in these fields require knowledge of functions. Therefore, the main purpose of this unit is to define and study functions and the methods of representing functions.

Objectives

When you have mastered this unit you should be able to:

1. Define and give examples of the following:
   a. function, domain, image and range
   b. functional notation
   c. constant function
   d. linear function
   e. exponential function
   f. polynomial function
   g. degree of a polynomial
   h. integral polynomials
   i. logarithm function
   j. common logarithm
   k. natural logarithm

l. scientific notation

m. characteristic of a logarithm

n. mantissa of a logarithm

o. antilogarithm.

2. Determine whether a functional relationship exists between two sets.

3. Specify the domain and range of a given function.

4. Describe functional notation and write a complete description of a given function.

5. Be able to produce the graph of a given function.

6. Describe and represent in graphic form an exponential function.

7. Be able to work with polynomials, performing operations of addition, subtraction, multiplication, division, factoring, and simplification on them.

8. Describe the technique of synthetic division and use it to solve problems.

9. Use the Remainder Theorem, Factor Theorem, Rational Zero Theorem, and Integral Zero Theorem to solve problems.

10. Write exponential equations that correspond to logarithmic equations.

11. Graph logarithmic equations.

12. Be able to state in writing the three "Laws of Logarithms" and solve problems using them.

Procedure

1. Read sections 4-1 through 4-4 in the text twice through - once quickly for an overview and then again for a more indepth study of

the material. Be able to give the definition of a function and its component parts - domain, image and range.

An important feature of this unit is stressed in section 4-2, that of functional notation. Read this quite carefully and be able to use functional notation to set up problems such as those assigned in the exercises.

In section 4-3 follow through closely the sample problems involving graphs of functions. Be able to graph functions.

In section 4-4 be able to give the definition of exponential function. Be able to graph an exponential function.

Throughout these four sections the text refers to such terms as "absolute-value function," "constant function," "squaring function," "area function," "distance function," "power function," etc. Think over in your mind how these terms interrrelate with the concepts you studied in Units 1 and 2. Be able to define and use these terms in problem-solving such as those assigned in the exercises. Notice how you are building one concept upon another into a structured system.

Solve problems: a) 5, 7, 9, 11 and 15 of exercises 4-1
b) 1, 3, 5, 9 and 11 of exercises 4-2
c) 1, 3, 5 and 9 of exercises 4-3
d) 1, 3, 5 and 7 of exercises 4-4.

2. Read sections 5-1 through 5-5 in the text through twice - once for an overview and a second time more carefully for detail. Take note of the examples of different kinds of polynomial functions and be able to differentiate between them (see pp. 119 & 135 in the text).

Be able to give a definition for "degree n." Work through the sample problems in section 5-2 and be able to give definitions for

"rational expression," "factor," and "remainder."

Section 5-3 presents division of a polynomial by x-a (where "a" is a constant) called synthetic division, and the illustrations and sample problems should be carefully followed to gain a clear understanding of the procedure. Be able to do problems such as those given in the exercise section.

Section 5-4 utilizes synthetic division to evaluate a polynomial function through two theorems - the Remainder Theorem and the Factor Theorem. Follow through the sample problems for these theorems to gain insight into their application. Be able to use these theorems to solve problems such as those in the exercises.

In section 5-5 the "integral zeroes" of a polynomial are explored. The principle behind this is that if you can find one "zero" of a polynomial through synthetic division, then the polynomial can be reduced into two factors and the larger factor can be explored for "zeroes" and then broken down into factors and so on until a polynomial of degree n is broken down into n factors and its roots derived from these n factors. Follow through the sample problems and be able to use the Integral Zero Theorem and the Rational Zero Theorem in problem-solving.

Solve problems: a) 1, 3, 5, 7, 9, 13 and 15 of exercises 5-1.
b) 1, 3, 5, 7, 9 and 11 of exercises 5-2.
c) 1, 3, 5, 7 and 9 of exercises 5-3.
d) 1, 3, 5, 7 and 9 of exercises 5-4.
e) 1, 3, 5, 7, 13, 17 and 19 of exercises 5-5.

3. Read sections 6-1 through 6-3 twice through - once for an overview and then again for more detail. Be able to give a definition for "logarithm function." Be able to construct logarithmic graphs such as those shown in section 6-2. In section 6-3 see how the Laws of Exponents, that you studied previously in Unit 1, are assumed to be valid for all real numbers and are used in the development of the three Laws of Logarithms.

Follow through the sample problems in these three sections in a step-by-step fashion. Be able to solve problems of a similar nature such as those given in the exercises.

Solve problems: Solve the odd-numbered problems in the "Review for Section 6-1 through 6-3" to gain a better working knowledge of logarithms before proceeding to the next sections.

4. Read section 6-4 and 6-5 twice through - the first time for an overview and the second time for more detail and depth.

In section 6-4 be able to describe the difference between "natural logarithms" and "common logarithms." Also, be able to give the definition for the terms "scientific notation," "characteristic" and "mantissa."

In section 6-5 you will learn how to approximate N if you know log N, then you will learn how to compute with common logarithms. Know the definition: "if M = log N then N is the 'antilogarithm' of M" and be able to use if in problem-solving. Follow through the sample problems in which roots, products and quotients are actually computed. Be able to use the Table of Common Logarithms in Appendix B of the text to solve problems similar to those in the exercises.

Solve problems: a) 1, 3, 5, 7, 9, 11, 13 and 15 of exercises 6-4.

b) 1, 3, 7, 11, 13, 15, 17, 19 and 21 of exercises 6-5.

5. Read the Summary paragraph, then take the self-test to see if you have mastered the objectives of the unit. If you have any questions or uncertainties, check with your proctor. When you have completed the self-test ask your proctor to administer the Unit 3 Mastery Test.

Summary

In this unit you have studied functions and some of their uses in algebra. You have learned new terms and concepts that will be of value to you in future units. With these new concepts and definitions, coupled with the problem-solving you have done in this unit, you now have a clearer understanding of the role of functions and their place in algebra. You should now proceed to the Unit 3 Self-Test to check your mastery of the concepts presented in this unit.

Unit 3 Self-Test

1. Graph each of the following functions and give the domain.

a. $f(x) = \frac{1}{x+2}$ b. $f(x) = x^{1/2}$

2. The function f whose domain is the set of real numbers is defined as:

$$f(x) = \begin{cases} 1 \text{ if } x < 0 \\ 0 \text{ if } 0 \leq x \leq 3 \\ x \text{ if } x > 3 \end{cases}$$

Find

a. $f(-4)$ b. $f(0)$ c. $f(7/3)$ d. $f(\pi)$

3. Perform the indicated operations.

a. $(x^4 + 2x^3 + 15x + 3)-(2x + 5)$ b. $(x^3 + 2x^2 + 3x)(x^3 + x^2 + 2)$

c. $(x^3 - x^2 + 5)\div(x^2 + 1)$

4. Use synthetic division to find the quotient and remainder on dividing $-3x^3 + 2x^2 -1$ by $x + 1$.

5. Find the rational zeros of each of the following polynomials.

a. $2x^3 - 9x^2 + 7x + 6$ b. $x^3 + x^2 - x - 1$

6. Factor each of the following into as many integral polynomials as possible.

a. $x^3 + 6x^2 + 11x + 6$ b. $x^3 + 4x^2 - 7x - 10$

7. Solve each of the following equations for x.

a. $x = 2 \log_2 8$ b. $\log_{10} x = 1/2$

8. Graph each of the following logarithmic equations and name the x-intercept.

a. $y = \log_3 x$ b. $y = 2 \log_5 x$

9. Express the following numbers in scientific notation.

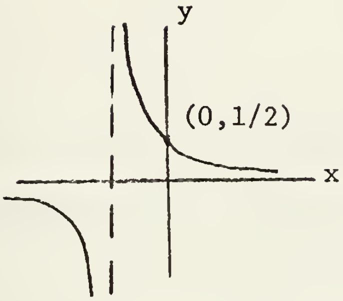
a. 587,000 b. .00876 c. 184.5

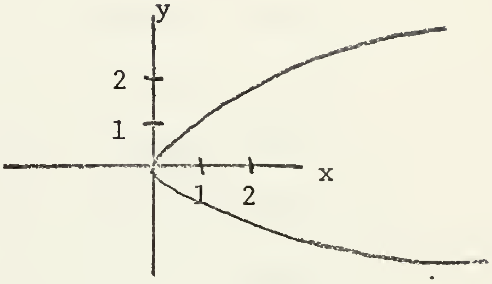
10. Use logarithms to find approximations to three significant digits for each of the following.

a. $\frac{487}{73}$ b. $8473 \times .088$ c. $\sqrt[3]{546}$

Answers to Unit 3 Self-Test

1. a. $D = \{x | x \neq -2\}$ b. $D = \{x | x \geq 0\}$





2. a. 1 b. 0 c. 0 d. π

3. a. $x^4 + 2x^3 + 13x - 2$ b. $x^6 + 3x^5 + 5x^4 + 5x^3 + 4x^2 + 6x$

c. $x - 1 + \frac{x+6}{x^2+1}$

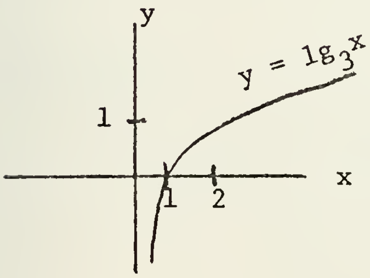
4. $(-3x^2 + 5x - 5)(x + 1) + 4$

5. a. 2, 3, -1/2 b. 1, -1, -1

6. a. $(x+1)(x+2)(x+3)$ b. $(x+5)(x+1)(x-2)$

7. a. 6 b. 4 c. 3

8. a.



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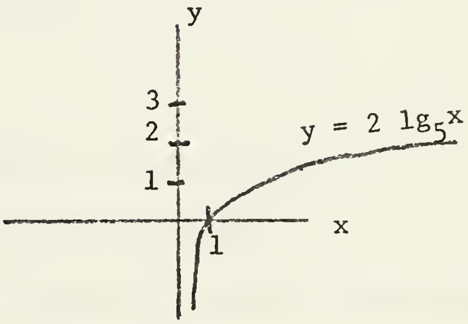


9. a. 5.87×10^5 b. 8.76×10^{-3} c. 1.845×10^2

10. a. 6.67 b. 745 c. 8.17

Unit 4

Topics in Trigonometry

Introduction

Do you know how to find the width of a river without swimming it? Do you know how to find the distance through the base of a mountain without tunneling through it? Do you know how to find the height of a flagpole without climbing it? Do you know how to find the measurement of an object without actually measuring it? Such problems must be solved by engineers constructing bridges, buildings, railroads; by astronomers in determining the time by which our clocks are regulated; and by a host of people involved in practical application of trigonometry everyday. This unit shows how such measurements may be determined. Trigonometry was originally developed as a study of the measurement of triangles, and is an especially fine tool for use in surveying. Trigonometry and the equations and identities connected with it requires a knowledge of algebra. You have had this necessary knowledge presented to you in Units 1, 2, and 3. This unit will be an extension of those ideas into the area of trigonometry.

Objectives

When you have mastered this unit you should be able to:

1. Define and give examples of the following:
    a. all six trigonometric functions (circular functions)
    b. the wrapping function
    c. quadrants
    d. periodic functions
    e. sines and cosines for every real number

    f. trigonometric identities
    g. radian and radian measure of an angle
    h. initial side, terminal side, and standard position for a general angle
    i. trigonometric cofunctions
    j. Law of Cosines
    k. Law of Sines
    l. the four trigonometric addition formulas
    m. double-angle formulas
    n. half-angle formulas
    o. period, amplitude and asymptotes for a trigonometric graph
    p. inverse trigonometric functions.

2. Find the trigonometric functions of a given angle and, conversely, find the angle given the numerical value of one of its trigonometric functions.

3. Compute the wrapping function, W, for a given coordinate in any quadrant and, conversely, find the coordinate of an indicated point given W.

4. Prove trigonometrical identities using the fundamental relations, double-angle and half-angle relationships.

5. Evaluate expressions such as $\frac{5\pi}{4}$.

6. Find the range of the basic trigonometric functions and describe the behavior of a given trigonometric function in a given interval.

7. Given the degree measure of an angle, convert it to radian measure and, conversely, convert radian measure to degree measure.

8. Convert functions to their respective cofunctions.

9. Solve right triangles and word problems involving right triangles.

10. Solve any triangle and problem involving such triangles using the Law of Cosines and the Law of Sines.

11. Find the exact value of trigonometric functions by using half-angle and double-angle formulas.

12. Recognize and solve trigonometric equations.

13. Graph trigonometric functions and describe their periods, amplitudes and asymptotes.

14. Solve problems involving inverse trigonometric functions and describe the graphs and domains of such functions.

Procedure

1. Read sections 7-1 through 7-3 in the text. Read the material twice through - once quickly for an overview and then again a second time more carefully making sure you can define the three trigonometric functions "sine," "cosine" and "tangent" given in section 7-1. Follow through the sample problems that make use of these functions and be able to use the Pythagorean Theorem.

In sections 7-2 and 7-3 be able to describe the wrapping function, its use, and its properties. Be able to give definition for "quadrant" and "period of a function." Follow through carefully the sample problems in these two sections. They are vital to the material and to your understanding of it.

Solve problems: a) 1, 3, 5, 7 and 9 of exercises 7-1.

b) 1, 7, 11 and 17 of exercises 7-2.

c) 1, 3, 5 and 7 of exercises 7-3.

2. Read sections 7-4 through 7-6 in the text. Read this material once for an overview of the subject matter and then again in greater detail to be able to define the terms "sine" and "cosine" and give their properties. Be able to describe the concept of trigonometric identities and be able to use them. Know the definitions for the other three trigonometric functions - secant, cosecant, and cotangent. Follow through the sample problems in these sections step-by-step. Be able to solve problem of a similar nature as those given in the exercises.

Solve problems: a) 1, 3, 5, 7, 11, 13 and 21 of exercises 7-4.

b) 1, 3, 7 and 13 of exercises 7-5.

c) 1, 3, 5, 7, 11 and 13 of exercises 7-6.

3. Read sections 7-7 and 7-8 in the text through twice - once for an overview and again in more detail. Be able to define "radian measure" and be able to convert radian measure to degree measure and vice versa. In section 7-8 the concept of cofunctions is explored. Follow through the sample to gain a clearer understanding of cofunctions and be able to solve problems involving cofunctions.

Solve problems: a) 1, 3, 5, 11, 13, 17, 19, 23, 27, 29, 31 and 33 of exercises 7-7.

b) 1, 3, 7, 9, 13, 15, 19, 21, 27 and 29 of exercises 7-8.

4. Read sections 8-1 through 8-3 in the text. Read it once through for an overview and a second time for more detail and a clearer understanding of the subject matter presented. See how the "Law of Cosines" and the "Law of Sinces" are used to solve for unknown parts in any triangle. Be able to state these laws in writing. Be able to solve problems using these laws.

Follow through the sample problems to see just how these laws are used in practical problem-solving. Be able to use the Law of Sines and logarithms to solve problems similar to those assigned in the exercises.

Solve problems: a) 1, 3, 7, 11, 13 and 15 of exercises 8-1.

b) 1, 3, 9, 11 and 13 of exercises 8-2.

c) 1, 3, 7, 11 and 15 of exercises 8-3.

5. Read sections 8-4 and 8-5 on addition formulas, half-angle formulas, and double-angle formulas. Read it twice through - once for an overview of the material and then a second time for a more indepth understanding of the definitions and concepts. Be able to state these definitions in writing.

Follow through the sample problems in these two sections in a step-by-step fashion. Be able to solve problems of this nature such as those given in the exercises.

Solve problems: a) 1, 5, 7, 11, 15, 17 and 21 of exercises 8-4.

b) 1, 3, 5, 11, 13 and 17 of exercises 8-5.

6. Read sections 8-6 through 8-8 on trigonometric equations and their graphs. Read it once through quickly for an overview and then again in more detail to see how these sections interrelate with themselves and all the concepts presented in this unit. Follow through the sample problems and be able to define "trigonometric equation," "asymptotes," and "amplitude," and "period."

Then solve problems: a) 1, 3, 7, 9, 11 and 13 of exercises 8-6.

b) 1, 3, 5, 7 and 9 of exercises 8-7.

c) 1, 3, 5, 7, 9, 15 and 17 of exercises 8-8.

7. Read section 8-9 through twice - a first time for an overview of the subject matter and then a second time for a greater depth. Be able to give definitions for "Arccosine," "Arcsine," and "Arctangent." Follow through the sample graphs and problems to gain a clearer insight into the subject matter. Be able to solve similar problems such as those given in the exercises.

Solve problems: 1, 3, 7, 9, 11, 15 and 17 of exercises 8-9.

8. Read the Summary paragraph, then take the self-test to see if you have gained a clear understanding and a working knowledge of the concepts of this unit. If there are any questions or uncertainties, check with your proctor. When you have completed the self-test ask your proctor to administer the Unit 4 Mastery Test.

Summary

In this unit you have studied trigonometric functions and related concepts. You have learned the definition of many terms, and through problem-solving, you have come to a clearer understanding of these terms and the concepts involved. This unit, as with previous units, is a link in a chain that builds one upon another to give you the insight and mastery necessary to be proficient in mathematics.

Unit 4 Self-Test

1. Find each number.

a. $\sin(\frac{-3\pi}{2})$ b. $\cos(\frac{3}{2}\pi)$ c. $\tan(-4\pi)$

d. Csc 210° e. Sec 135° f. Cot 225°

2. Given that W( ) = P, determine the coordinate (x,y) of P.

3. Simplify each of the following.

a. $\sec x \cos x - (1-\sin^2 x)$ b. $\frac{1+\tan^2 x}{\cot^2 x+1}$

4. Prove the following identities.

a. $2\csc x = \frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$ b. $\frac{\sec A - \csc A}{\sec A + \csc A} = \frac{\tan A - 1}{\tan A + 1}$

5. Find the value of the following trigonometric functions. Convert radian measure to degree measure.

a. $\cos\frac{7\pi}{6}$ b. $\sin(\frac{-\pi}{3})$ c. $\mathrm{CoTan}(\frac{-3}{2}\pi)$

6. Express each of the following in terms of a trigonometric function of x.

a. Sin(90°+x) b. Sin(-x) c. $\tan(\pi+x)$

7. Triangle ABC has sides a, b, and c, and angles opposite these sides of A, B, and C respectively.

a. If C = 25, <A = 35°, <B = 68° then find the remaining parts.

b. If C = 628, b = 480, and LC = 55° then find the remaining parts.

c. If a = 132, b = 224, and LC = 28° then find the remaining parts.

8. Use the half-angle formula to find the sine, cosine and tangent of $\pi/12$.

9. Use the double-angle formula to find the sine, cosine and tangent of $2\pi/3$.

10. Simplify:

a. $\text{Sin}2x\text{Tan}x + \text{Cos}2x$

b. $\frac{\text{Cos}2x}{\text{Cos}x+\text{Sin}x}$

11. Find the solutions set for the indicated interval.

a. $(\text{Tan}x-1)(4\text{Sin}^2x-3) = 0$ for $0 \leq x \leq 2\pi$

b. $\text{Csc}x + \text{Cot}x = \sqrt{3}$ for $0 \leq x \leq 2\pi$

c. $\text{Sin}^2x + \text{Sin}x - 2 = 0$ for $0 \leq x \leq 2\pi$

12. State the period and amplitude for the following trigonometric functions and draw the graph.

a. $y = 4\text{Sin}x$

b. $y = 3\text{Cos}\,\frac{1}{2}x$

c. $y = \text{Sin}x + \text{Cox}$

13. Find each of the following.

a. arcSin 2

b. arccos 1

c. arctan 1

d. arcsec 2

e. $\text{Sec}[\arcsin(-\frac{1}{2})]$

f. $\text{Sin}[\arcsin \frac{1}{2}]$

14. Prove the following identities.

a. $\frac{\text{Sin}4x + \text{Sin}2x}{\text{Cos}4x + \text{Cos}2x} = \text{Tan}\,3x$

b. $\text{Tan}\,x\,\text{Sin}2x = 2\,\text{Sin}^2x$

c. $\frac{1 + \text{Cos}2x}{\text{Sin}2x} = \text{Cot}\,x$

Answers to Unit 4 Self-Test

1. a. -1 b. 0 c. 0 d. -2 e. $\frac{-2\sqrt{3}}{3}$ f. 1

2. $(1/2\sqrt{2}\ ,\ 1/2\sqrt{2})$

3. a. Sin^2x b. Tan^2x

4. a. $$2\text{Cscx} = \frac{\text{Sinx}}{1+\text{Cosx}} = \frac{1+\text{Cosx}}{\text{Sinx}}$$

$$= \frac{\text{Sin}^2x + (1+\text{Cosx})^2}{\text{Sinx}(1+\text{Cosx})}$$

$$= \frac{\text{Sin}^2x + 1 + 2\text{Cosx} + \text{Cos}^2x}{\text{Sinx}(1+\text{Cosx})}$$

$$= \frac{2 + 2\text{Cosx}}{\text{Sinx}(1+\text{Cosx})} \quad \frac{2(1+\text{Cosx})}{\text{Sinx}(1+\text{Cosx})} = \frac{2}{\text{Sinx}}$$

$$= 2\text{Csc}\ x$$

b. $$\frac{\text{SecA}-\text{CscA}}{\text{SecA}+\text{CscA}} = \frac{\text{TanA}-1}{\text{TanA}+1}$$

$$\frac{\text{SecA}-\text{CscA}}{\text{SecA}+\text{CscA}} = \frac{\frac{1}{\text{CosA}} - \frac{1}{\text{SinA}}}{\frac{1}{\text{CosA}} + \frac{1}{\text{SinA}}}$$

$$= \frac{\frac{\text{SinA}}{\text{CosA}} - 1}{\frac{\text{SinA}}{\text{CosA}} + 1} = \frac{\text{TanA} - 1}{\text{TanA} + 1}$$

5. a. $\frac{-\sqrt{3}}{2}$ b. $-\sqrt{3}$ c. 0

6. a. Cos x b. -Sin x c. Tan x

7. a. $<C = 77°$, $a = 15$, $b = 24$

8. a. Sine $\frac{\pi}{12} = .259$ b. Cos $\frac{\pi}{12} = .966$ c. Tan $\frac{\pi}{12} = .268$

9. a. Sin $\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ b. Cos $\frac{2\pi}{3} = -\frac{1}{2}$ c. Tan $\frac{2\pi}{3} = -\sqrt{3}$

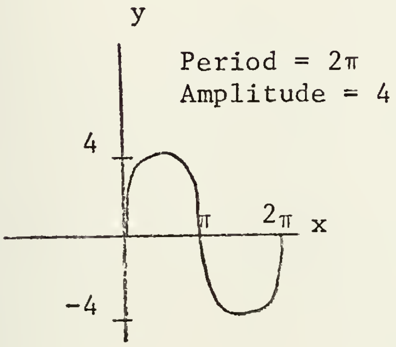
10. a. Cos x − Sin x b. 1

11. a. $x = \pi/4, \pi/3, 2\pi/3, 5\pi/4, 4\pi/3, 5\pi/3$

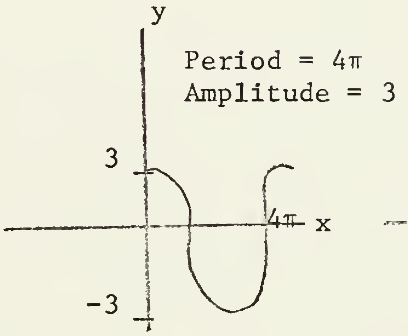
b. $x = \pi/3$, other roots extraneous

c. $x = \pi/2$

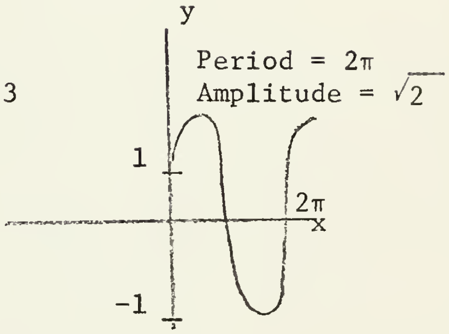
12. a.



b.



c.



13. a. does not exist b. 0° c. $\pi/4$

d. $\pi/3$ e. $\frac{2\sqrt{3}}{3}$ f. 1/2

14. a. $\frac{\text{Sin}4x + \text{Sin}2x}{\text{Cos}4x + \text{Cos}2x} = \text{Tan } 3x$

$\frac{2 \text{ Sin } 1/2(4x+2x) \text{ Cos } 1/2(4x-2x)}{2 \text{ Cos } 1/2(4x+2x) \text{ Cos } 1/2(4x-2x)} = \text{Tan } 3x$

$\frac{\text{Sin } 3x}{\text{Cos } 3x} = \text{Tan } 3x$

$\text{Tan } 3x = \text{Tan } 3x$

b. $\text{Tan}x \text{ Sin}2x = 2\text{Sin}^2x$

$\frac{\text{Sin } x}{\text{Cos } x} \cdot 2 \text{ Sin}x \text{ Cos}x = 2 \text{ Sin}^2x$

$$2 \operatorname{Sin}^2 x = 2 \operatorname{Sin}^2 x$$

c. $\frac{1+\operatorname{Cos} 2x}{\operatorname{Sin} 2x} = \operatorname{Cot} x$

$$\frac{1+2\operatorname{Cos}^2 x-1}{2\operatorname{Sin}x\operatorname{Cos}x} = \frac{\operatorname{Cos}x}{\operatorname{Sin}x}$$

$$\frac{2\operatorname{Cos}^2 x}{2\operatorname{Sin}x\operatorname{Cos}x} = \frac{\operatorname{Cos}x}{\operatorname{Sin}x}$$

$$\frac{\operatorname{Cos}x}{\operatorname{Sin}x} = \frac{\operatorname{Cos}x}{\operatorname{Sin}x}$$

IV. EVALUATING THE EFFECTIVENESS OF TEACHING MA 1021 IN THE PSI MODE

In the past mastery of a subject has been considered possible for only a minority of students, and the grading system has been adjusted to reflect this assumption. Mastery of a subject under this type of grading system is unattainable for students, but this is because the system is set up to portion out only so many A's, B's, etc., no matter how well the student performs or how much better the "B" is from previous "A's." According to Bloom (3), mastery must be both subjective and objective in recognition. Subjectively the student must have the feeling of control over ideas and skills. The student must come to recongize that he "knows" and can do what the subject requires. Objectively he must be evaluated with a grade. Bloom states:

> "If the system of formative evaluation (diagnostic-progress tests) and the summative evaluation (achievement examinations) informs the student of his mastery of the subject, he will come to believe in his own mastery and competence. He may be informed by the grading system as well as by the discovery that he can adequately cope with the variety of tasks and problems in the evaluated instruments."

When the student has mastered a subject he receives both subjective and objective evidence of such mastery and there is a change in the student's view of himself and his environment. Bloom further contends that the student begins to like the subject and to desire more of it. The student desires some feeling of control over his environment and mastery of a subject gives him such a feeling. Interest in a subject is both a cause of mastery of the subject as well as a result of such

mastery, and motivation of further learning is an important result of mastery.

A. RESULTS

How effective is the PSI method of learning? To date we have limited evidence, but according to Bloom (3), the results are very encouraging. In a subject where he was able to utilize comparable achievement tests for a subject in 1965, 1966 and 1967, 20 percent of the students received a grade of A on the final examination in 1965. This was before the PSI strategy was employed. In 1966, after employing the PSI strategy, 80 percent of the students attained the same level of mastery in the parallel examination and were given the grade of A. The difference in the mean performance of the two groups represented about two standard deviations on the 1965 achievement test, and this, according to Bloom, is highly significant.

In 1967 Bloom used the same formative evaluation tests he used in 1966. It was possible to compare the 1966 and 1967 results after each unit of learning, therefore, the formative evaluation tests became quality control measures. Where there were significant negative difference between the results on a particular test from 1966 to 1967, the instructor reviewed the specific learning difficulties and explained the ideas in a different way. The final results on the 1967 evaluation was that 90 percent of the students achieved mastery and were given grades of A.

Bloom further states that similar studies are being performed at different levels of education in which he expects failures and successes, the point being that no single strategy of learning mastery can be used rotely to achieve a particular set of results. The problem is rather one

of determining what procedures will prove most effective in assisting individual students to master the subject matter under consideration.

B. RESULTS AT NPS

A carefully controlled experiment in MA 2045 (Computational Linear Algebra) was designed and conducted by Professor Maurice Weir for the purpose of determining the relative instructional effectiveness of PSI at NPS. Professor Weir's findings were subsequently reported in a Master's Thesis by Captain Patrick A. Toffler (12). The course study guide was revised following each trial, field tested twice, and subjectively established as valid. Next, two groups of students (12 in PSI, 16 in class-lecture-recitation method (CLR)) were randomly selected by the Registrar at NPS and instructed by the same professor. The two groups were given the linear algebra course and the performance of each group was measured and the two groups compared with respect to the following variables which were subjected to several statistical tests:

1. Percentage of students completing the course: All students completed both courses. This is a requirement at NPS.

2. Final examination scores:
    a. A two-hour closed-book examination covering all course material was given to both groups.
    b. The examination was written by the course instructor in collaboration with the Chairman of the Mathematics Department, reviewed by the tutors and other mathematics professors.
    c. The same grading scheme was used for both PSI and CLR.
    d. PSI students scored significantly higher than the CLR

students ($p = .005$). (Ref. pp. 91-93 Toffler's Thesis).

3. Course grades were computed in a manner described on pp. 52-53 of Toffler's Thesis. The mean grade of PSI students was numerically and statistically higher than the CLR group according to the univariate F-ratio test ($p = .025$).

4. Rate of progress: Over 50% of the PSI students completed the course two weeks before the end of the term.

5. Facilitation: The purpose of this test was to measure the effect of PSI on the performance of students in their other CLR courses at NPS. In order to obtain this measure Toffler compared grade point averages (GPA) of both groups for the quarter with the effect of the MA 2045 grade removed. Again the PSI group performed significantly better than the CLR group ($p = .075$). (See pp. 57-58 and pp. 97-98 of Toffler's Thesis).

6. Student attitudes: A questionnaire was designed and administered to both PSI and CLR students (see pp. 102-105 of Toffler's thesis for questionnaire). Toffler's conclusion was that the PSI mode generated within its students certain attitudes that tended to be more positive than those exhibited within the CLR group (refer to pp. 60 of Toffler's Thesis).

In addition to the linear algebra course, there have been other courses taught using the PSI method at NPS, notably Commander Joseph Cyr's course in Networks, Flows and Graphs (OA 4633) and Professor John Dyer's course in Waves and Particles (Physics 2251). At the completion of Commander Cyr's course the students (8 in PSI, 4 in CLR) were compared across the output measures generated by the final examination, the rate of progress, and the student attitude questionnaire. The results were in

complete agreement with those found in the main experiment discussed in the Toffler thesis.

In Professor Dyer's course the students were all taught using PSI. The course has been in use for two quarters and Professor Dyer states that he has had good results and that students have expressed that they have better retention under PSI. Professor Dyer feels that weaker students have done much better in the PSI mode than the CLR mode.

C. IMPLICATIONS FOR MA 1024

As concluded by Toffler (12), it is valid to extrapolate beyond the linear algebra course and to apply the results of his study to similar courses of similar scope in other areas of mathematics. The course in MA 1024 (pre-calculus mathematics) is one such course.

It is recommended that the PSI course developed in this thesis be tested at NPS under conditions similar to those under which linear algebra was conducted and tested.

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c.1 The development of a
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for pre-calculus
mathematics (MA 1021).